Marginal Sharpe Ratio

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Aug 22, 2024, (Latest Revision Sep 14, 2024)

Abstract

The Marginal Sharpe Ratio (MSR) of an investment strategy with respect to a total portfolio can be defined as the derivative of the total portfolio Sharpe ratio (SR) over the allocation weight. Defined in such a way, MSR takes into account not only the contribution of the new strategy to the portfolio expected returns, but also the expected change of the portfolio risk profile due to diversification. It is similar to the well known concept of marginal risk. In this paper we derive analytical expressions for MSR. This leads to the very simple and intuitive formulas, such as for example, $\texttt{MSR} = \frac{\sigma}{\sigma_p}\texttt{SR} - \beta \cdot \texttt{SR}_p$. Here $\texttt{SR}$ and σ refer to the SR and the standard deviation of returns for the individual strategy. $\texttt{SR}_p$ and σ_p refer to the SR and the standard deviation of returns for the base portfolio. β refers to the beta coefficient of the linear regression of the strategy returns against the total portfolio returns. The formula can be used for quick, "back-of-envelope" calculations for appraisals of new investment opportunities taking into account the risk/return profile of a base portfolio. The use of the MSR formula is demonstrated with simple numerical experiments.

1 Introduction

The Sharpe ratio (SR) [1] is one of the most popular measures for assessing performance of individual investment strategies. Formally, it is defined as the expected excess return per unit of risk, i.e., a ratio between expected return and standard deviation of returns.

$$\texttt{SR} = \frac{\mu - r_f}{\sigma} \tag{1}$$

where μ is the expected return, r_f is the risk free rate, and σ is standard deviation of returns. In practice it is often reported in annualized terms and under the assumption of $r_f = 0$.

When comparing two individual investment strategies with equal expected risk, a rational investor would prefer the strategy with higher expected return. Similarly, when comparing two investment opportunities with equal expected returns, a rational investor would prefer the investment with lower expected risk.

The Sharpe ratio is just one of the metrics in the family of performance ratio measures. There is no perfect measure of performance, and SR has its limitations [2, 3, 4, 5, 6]. However, it is a useful metric and very popular one in the investment industry [7]. There are alternative measures of risk that can be used in the denominator which leads to alternative and popular performance ratio measures such as: Sortino, Traynor, Recovery, Calmar and Sterling. This document is focused on the Sharpe ratio only, but the approach proposed in this paper can be extended to other choices of performance ratios.

*Any opinions, findings, and conclusion or recommendations expressed in this material are those of the author and do not necessarily reflect the view of the Abu Dhabi Investment Authority.

Consider now a very typical situation in the investment management industry where a portfolio manager, running a big investment portfolio, is assessing new investment opportunities [8, 9]. As an example, this can be a Chief Investment Officer of a hedge fund running a big portfolio of systematic trading strategies. Suppose that she/he is now assessing several new strategies. Which one, if any, should be added to the existing portfolio? How to sort or prioritize the investment opportunities? As a second example, this can be a Portfolio Manager of a pension fund, who is assessing a list of new asset managers and investment opportunities where the pension money can be invested. What is the best way compare the investment opportunities at hand?

The performance of the individual investment opportunity i can be characterized with the individual Sharpe ratio $\texttt{SR}_i$. Although this can be a useful measure for an initial and quick assessment, it is important to realize that the individual performance measures are not sufficient. It is necessary also to assess how would the expected performance of the total portfolio change after adding the new investment. The new additions to the portfolio are expected to change both the expected returns and the risk of the total portfolio. Some investments might add little of expected gain but could be very favorable if they significantly decrease the risk of the total portfolio. It might be even preferable to accept a strategy with a negative individual SR, if it reduces the total risk (denominator in 1) to such an extent that the total portfolio SR is increased.

The importance of considering how a new investment might change a risk profile of a portfolio is well known by investment professionals. To quantify contributions of specific investments to the total portfolio risk one can use Marginal Risk (MR) measure (see Ref: [10, 11]). In this paper we present a measure which considers contributions of specific investment opportunities to the Sharpe Ratio of a total portfolio. The proposed approach considers not only the changes of the total risk profile as MR does, but also considers the changes of expected return.

In order to assess new investment opportunities one can consider the incremental change of SR, i.e., the Incremental Sharpe Ratio (ISR):

$$\Delta\texttt{SR} = \texttt{SR}_{final} - \texttt{SR}_{base} \tag{2}$$

where $\texttt{SR}_{base}$ is the Sharpe ratio of the base portfolio and $\texttt{SR}_{final}$ is the Sharpe ratio of the final portfolio. $\Delta\texttt{SR}$ can be easily calculated numerically, if we know exactly the target allocation for the new investment, along with the expected returns and the covariance matrix of the base portfolio and the new investment. The article [12] provides deeper analysis and analytical results which can be used in practice. Moreover, there are many software tools for portfolio optimizations and portfolio analytics which are available to investment practitioners. For a PM it is a relatively simple and routine task to calculate incremental changes of SR, and even to calculate optimal weights for the new target portfolio taking into account expected returns, risk and various other trading constraints.

However, there are situations where the PM needs to perform quick assessments using a limited information on performance statistics of individual strategies and without defined target allocations for the new investments. Additionally, from a practical point of view, it is optimal to divide the capital allocation process into two distinct phases: (1) new strategy assessment/approval and (2) portfolio optimization ([8]). In the first phase, the performance of the individual strategy is assessed using variety of methods (including SR) and the exact allocation to the new investment is yet unknown.

There is a mathematical object closely related to incremental SR change 2 which can help in this situation. It can be defined as the derivative of the portfolio Sharpe ratio over the strategy weight and we will call it a Marginal Sharpe Ratio (MSR).

$$\texttt{MSR} = \frac{\partial \texttt{SR}}{\partial w_i} \tag{3}$$

MSR measures how much would the Sharpe ratio of the total portfolio change, if one increases the weight w_i of a strategy (asset) i by a small amount. It considers not only the expected changes of the portfolio returns, but also expected changes of the total portfolio risk profile.

In the section 2 of this paper we will show that MSR can be derived analytically, leading to the very simple and intuitive formulas, such as for example, $\texttt{MSR} = \frac{\sigma}{\sigma_p}\texttt{SR} - \beta \cdot \texttt{SR}_p$. Here $\texttt{SR}$ and σ refer to the SR and the standard deviation of returns for the individual strategy. $\texttt{SR}_p$ and σ_p refer to the SR and the standard deviation of returns for the base portfolio. β refers to the beta coefficient of the linear regression of the strategy returns against the total portfolio returns.

The analytical formula for MSR can be very useful for quick, "back-of-envelope" calculations for appraisals of new investment opportunities assessed with respect to a base portfolio. Due to its simplicity the MSR can help in clear understanding what are the main drivers for changes of the overall performance in terms of the Sharpe Ratio measure. We will demonstrate this with the simple numerical examples in the section 3.

There are several articles discussing similar concepts such as [13, 14, 15]. However, those papers are defining the marginal Sharpe ratio in a different way and thus give us answers to different questions.

2 Derivation of a formula for Marginal Sharpe Ratio

Suppose that our investment universe consists of a list $[a_1, a_2, ..., a_M]$ of investment opportunities. An investment opportunity could be any investment asset, a particular financial instrument (such as stock or ETF for example), a systematic trading strategy, another portfolio, etc. In the further text we will always refer to investment strategies, or strategies in short.

A portfolio can be represented by a vector of dollar allocations $[v_1, v_2, ..., v_M]$, where $v_i = n_i p_i$ are "dollar" values invested into each strategy,, with p_i representing price/value of unit investment and n_i representing number of units invested. Alternatively, the portfolio can be represented with a list of the strategy weights $[w_1, w_2, ..., w_M]$, where $w_i = \frac{v_i}{\sum_i v_i}$.

The portfolio return on a day t is:

$$r_t^{(p)} = \sum_i w_{i,t-1} r_{i,t} \tag{4}$$

where $r_{i,t}$ is the return of strategy i at time t, and $w_{i,t-1}$ is a weight of the strategy i at the previous day $(t-1)$.

The expected portfolio return is a function of the strategy weights:

$$\mu_p = \sum_i w_i \mu_i \tag{5}$$

where μ_i is expected return of a strategy i

$$\mu_i = E[r_i] \tag{6}$$

The standard deviation of returns is one of the measures for the risk of the portfolio P, and it can be expressed as:

$$\sigma_p(w_1, ..., w_M) = \sqrt{\sum_{i,j} w_i w_j \rho_{i,j} \sigma_i \sigma_j} \tag{7}$$

Note that both expected return and standard deviation of portfolio returns are functions of allocation weights.

The main question of this article is how to evaluate an investment strategy, taking into account the base portfolio P. What criteria should we use in order to decide on which strategies are more or less attractive?

It is natural to consider a marginal improvement to the total portfolio Sharpe ratio, due to the addition of the new strategy i, as

$$\texttt{MSR}_i = \frac{\partial \texttt{SR}_p}{\partial w_i} \tag{8}$$

where $\texttt{SR}_p$ is the Sharpe ratio of the total portfolio. This object that tells us how much would the Sharpe ratio of the total portfolio change if we increase the weight w_i for a small amount. It takes into account the returns of a strategy but also diversification effects! We are going to call it a Marginal Sharpe Ratio (MSR).

The concept of MSR is similar to the well known concept of marginal risk (MR) $\texttt{MR}_i = \frac{\partial \sigma_p}{\partial w_i}$. MR can be helpful in identifying main risk contributors and providing more balanced portfolios from the risk perspective [10, 11]. See the Appendix 7 for a brief overview of MR.

We should note that increasing leverage, by multiplying all weights with the same factor, does not change the Sharpe ratio of the whole portfolio. We can therefore say that SR is a homogeneous function of the weights, of the order 0. The theory of homogeneous functions are briefly introduced in the Appendix 6. Following this observation we can apply the Euler theorem to SR which leads to:

$$\sum_i w_i \frac{\partial \mathtt{SR}_p}{\partial w_i} = 0 \tag{9}$$

As we can see, MSR appears naturally in the equation 10. Each term in the sum contains the MSR value for each strategy, multiplied by the strategy weight, and we can rewrite the equation 10 as:

$$\sum w_i \mathtt{MSR}_i = 0 \tag{10}$$

Some strategies contribute positively to the portfolio SR, some strategies contribute negatively, with the total weighted sum of the individual MSR contributions equal to zero.

Let's derive analytical expression for MSR. We start with SR definition

$$\mathtt{SR}_p = \frac{\mu_p}{\sigma_p} \tag{11}$$

After combining the equations 11 and 8 we get the expression for portfolio MSR

$$\mathtt{MSR}_i = \frac{\partial}{\partial w_i}\left(\frac{\mu_p}{\sigma_p}\right) \tag{12}$$

The derivative of the nominator is

$$\frac{\partial}{\partial w_i}\mu_p = \mu_i = \sigma_i \mathtt{SR}_i \tag{13}$$

The derivative of the denominator is

$$\frac{\partial}{\partial w_i}\left(\frac{1}{\sigma_p}\right) = -\frac{1}{\sigma_p^2}\frac{\partial}{\partial w_i}\sigma_p \tag{14}$$

It can easily be shown that

$$\frac{\partial}{\partial w_i}\sigma_p = \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p} \tag{15}$$

After importing the equations 13, 14 and 15 into 12 we get the final expression for MSR

$$\mathtt{MSR}_i = \frac{\sigma_i}{\sigma_p}\mathtt{SR}_i - \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p^2}\mathtt{SR}_p \tag{16}$$

The MSR formula 16 can be expressed in several alternative and intuitive ways. Instead of the covariance we can use the beta of the strategy i with respect to the base portfolio $\beta_{i,p} = \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p^2}$. In this way we get

$$\mathtt{MSR}_i = \frac{\sigma_i}{\sigma_p}\mathtt{SR}_i - \beta_{i,p}\mathtt{SR}_p \tag{17}$$

The first term is the SR of the individual strategy, re-normalized by the ratio of the strategy's and base portfolio's volatilities. Strategies with higher volatilities, i.e., with increased leverage contribute more to the overall portfolio's SR. This needs to be adjusted for diversification effects by subtracting the product of the strategy beta and the SR of the base portfolio. Strategies with positive betas lead to decreased MSR, whereas the strategies with negative betas lead to increased MSR.

Using the correlation $\rho_{i,p} = \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p \sigma_i}$ we can get a second alternative and simple expression:

$$\mathtt{MSR}_i = \frac{\sigma_i}{\sigma_p}\left(\mathtt{SR}_i - \rho_{i,p}\mathtt{SR}_p\right) \tag{18}$$

The formula for MSR can also be expressed with the use of Marginal Risk (see the Appendix 7))

$$\mathtt{MSR}_i = \frac{\mu_i - \frac{\mathtt{MR}_i}{\sigma_p}\mu_p}{\sigma_p} \tag{19}$$

It shows that MSR can be considered as a ratio of the adjusted expected return and standard deviation of returns. Expected return of the strategy i is adjusted by its marginal risk contribution. The contribution of the strategy i to the overall SR is positive when $\mu_i > \frac{\mathtt{MR}_i}{\sigma_p}\mu_p$

Imagine that the $\texttt{SR}_i > 0$ and $\texttt{SR}_p > 0$. If the strategy i is positively correlated to the base portfolio, the marginal Sharpe ratio of the strategy will be lower than the individual SR, i.e.. If marginal risk contribution is negative, i.e., if the addition of asset i decreases the overall portfolio risk, the adjustment is positive!

The formula also shows that even the strategy with zero or negative expected return $\mu_i <= 0$ can increase the overall Sharpe ratio of the portfolio if it decreases the risk of the overall portfolio. That can happen when the marginal risk contribution is negative and $\texttt{MR}_i < \frac{\mu_i \sigma_p}{\mu_p}$ (assuming $\mu_p > 0$).

All expressions are quite simple and intuitive. All of the shown formulas are presenting the same picture in different ways: MSR is a difference between volatility adjusted SR of the individual strategy and a risk diversification term.

Of course, MSR is closely related to ISR (Eq: 2). It can be easily calculated numerically by increasing the weight i for a small amount $w_i \to w_i + \Delta w_i$, and calculating resulting incremental change of the portfolio SR

$$\texttt{MSR}_i = \frac{\Delta \texttt{SR}}{\Delta w_i} \tag{20}$$

3 Numerical experiments

In this section we are going to demonstrate the application of the marginal Sharpe ratio using synthetic data with realistic parameters.

In the first example, suppose that Portfolio Manager (PM) has a large investment V in the base portfolio P. The expected Sharpe ratio is $\texttt{SR}_p = 1$ and the standard deviation of returns is $\sigma_p = 0.1$. Now PM is considering investing additional and smaller amounts $v < V$ of cash into the three new strategies $i = 1, 2, 3$. For each strategy we have the expected SR, the standard deviation of returns and the correlation between the strategy and a base portfolio P. For the first strategy the estimates are $\texttt{SR}_1 = 2$, $\sigma_1 = 0.1$ and $\rho_1 = 0.7$. For the second strategy: $\texttt{SR}_2 = 1.5$, $\sigma_2 = 0.1$ and $\rho_2 = -0.2$. And for the third strategy: $\texttt{SR}_3 = 1.5$, $\sigma_3 = 0.2$ and $\rho_3 = -0.2$. All figures are in annualized terms.

Note that the best individual performance is expected from the first strategy which has the highest Sharpe ratio. Sharpe ratios for the second and the third strategy are equal. The third strategy is a "leveraged" version of the second, though. The risk of the third strategy is twice as much as of the second and the expected returns are two times higher. Therefore, if one looks at the individual performances only, the first strategy would be preferred over the second and third. The investor could be indifferent between the second and the third strategy, based on the individual Sharpe ratios only.

However, when we look at the marginal Sharpe ratios, the picture becomes very different. Using the equation 18 and the available estimates, the marginal Sharpe ratios can be easily calculated for all three investment opportunities. For example, for the first strategy we get $\texttt{MSR}_1 = \frac{0.1}{0.1}(2 - 0.7 \cdot 1) = 1.3$. Application of the formula 18 to all strategies leads to the following results: $\texttt{MSR}_1 = 1.3$, $\texttt{MSR}_2 = 1.7$ and $\texttt{MSR}_3 = 3.4$. When we consider diversification effects using MSR, the strategy 2 becomes preferable over the strategy 1. The strategy 1 has the highest SR, but is also positively correlated with the base portfolio. On the other hand, the strategy 2 is negatively correlated with the base portfolio, which helps in decreasing overall risk, leading consequently to higher marginal increase of the total portfolio SR. The strategy 3 is even more preferable than strategy 2, although the individual Sharpe ratios are equal. Strategy 3 is more leveraged and is expected to bring more gains, with risks diversified inside the base portfolio.

In the second example, lets us suppose that the PMs best estimates of future distribution of returns are given by historical, sample estimates of means, standard deviations and correlations. The Fig:1 is showing time series of cumulative returns of 6 strategies. The returns are generated using one-factor model with Gaussian innovations.

What matters for the application of MSR are: the strategy average returns, the standard deviations of the returns and the Sharpe ratio of the individual strategies which are shown in the table in Fig:2.

The Fig:3 shows the table with sample correlation matrix.

Suppose that the PM has a large investment V into

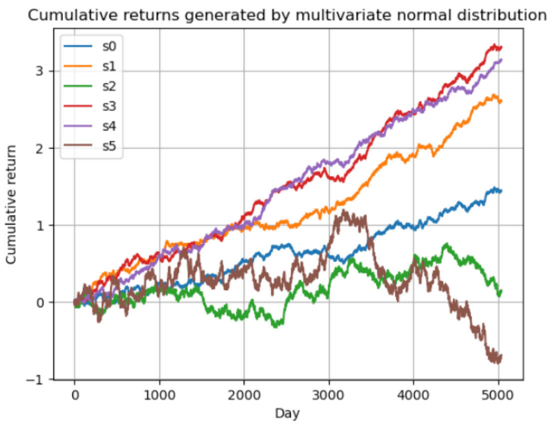

Figure 1: Cumulative returns generated with one-factor model with Gaussian innovations

	mean	std	sr
A0	0.07	0.1	0.72
A1	0.13	0.1	1.29
A2	0.01	0.2	0.04
A3	0.17	0.1	1.64
A4	0.16	0.1	1.57
A5	-0.03	0.3	-0.11

Figure 2: Means and standard deviation of synthetic strategies

	A0	A1	A2	A3	A4	A5
A0	1.00	0.31	-0.31	0.90	0.50	-0.80
A1	0.31	1.00	-0.09	0.28	0.17	-0.26
A2	-0.31	-0.09	1.00	-0.28	-0.14	0.23
A3	0.90	0.28	-0.28	1.00	0.45	-0.72
A4	0.50	0.17	-0.14	0.45	1.00	-0.40
A5	-0.80	-0.26	0.23	-0.72	-0.40	1.00

Figure 3: Correlation matrix of synthetic strategies

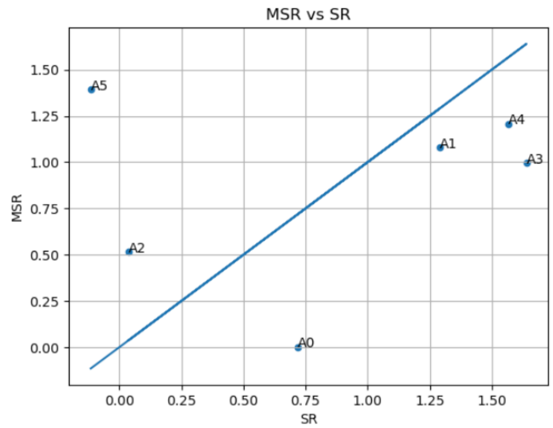

Figure 4: Scatter plot of MSR vs SR, with base portfolio represented by A0

the strategy A0. Now PM is considering investing additional and smaller amounts $v < V$ into some of the new opportunities represented by strategies [A1, A2, A3, A4 and A5]. SR for the base portfolio is modest at $\mathtt{SR}_0 = 0.72$. Using individual Sharpe ratios only, one could sort and prioritize the strategies in the following way [A3, A4, A1, A2, A5].

Let's see what MSR tells us about the investment opportunities. Using the parameters from the tables 2 and 3 we calculated MSR for each investment, assuming that the base portfolio corresponds to the asset A0. The chart 4 shows the scatter plot of MSR vs SR for all synthetic strategies.

We have plotted a 45-degree line representing the points where $\mathtt{MSR} = \mathtt{SR}$. If the point is above the line, that means that the strategy's MSR is higher than individual SR, which can be caused either by negative correlation to the base portfolio (diversification effect), or by the volatility of the individual strategy being higher than that of the base portfolio's (for example by increased leverage). On the other hand, the points below the 45-degree line represent the strategies which are either positively correlated to the base portfolio, or having lower volatility from the base portfolio.

Numerical results are presented in the table from Fig:5. The table shows marginal Sharpe ratios in the last column (MSR). It also shows marginal risks

	SR	mean	std	corr	beta	MR	MSR
A0	0.72	0.07	0.1	1.00	1.00	0.10	0.00
A1	1.29	0.13	0.1	0.31	0.31	0.03	1.08
A2	0.04	0.01	0.2	-0.31	-0.61	-0.06	0.51
A3	1.64	0.17	0.1	0.90	0.91	0.09	0.99
A4	1.57	0.16	0.1	0.50	0.50	0.05	1.20
A5	-0.11	-0.03	0.3	-0.80	-2.41	-0.24	1.39

Figure 5: Numerical results: MSR vs SR, with base portfolio represented by A0

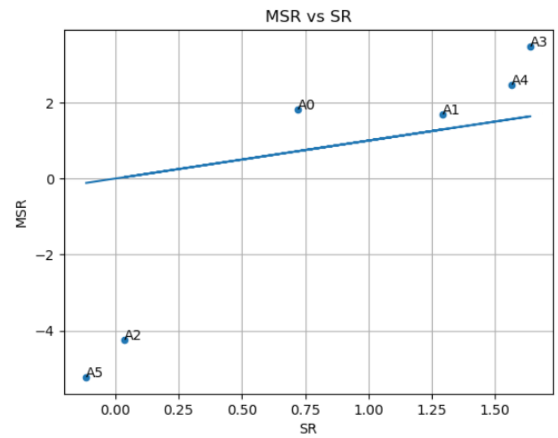

Figure 6: Numerical results: MSR vs SR, with base portfolio represented by equally weighted sum of all assets

(MR), along with the individual performance statistics and correlations (corr) with the base portfolio

These results imply that prioritization should be very different from the one implied by the individual performances. Marginal Sharpe ratios imply that the strategies should be sorted as [A5, A4, A1, A3, A2]. Although the individual Sharpe ratio of A5 is negative, due to the very negative correlation with the base portfolio A0, marginal Sharpe ratio is highest! Marginal risk (MR) for A5 is negative which means it is decreasing the risk of the total portfolio. Therefore the strategy A5 can be useful in hedging risk. The strategy A4 looks better than A3 even if individual SR of A3 is higher. This is because A4 is less correlated to the base portfolio. Marginal Sharpe ratio of A0 is zero, as SR does not change if we add more of a same asset.

Note that MSR depends on the composition of the base portfolio. In the previous example, we have supposed that the base portfolio is A0. Let's suppose now that the base portfolio is equally weighted portfolio of all assets [A0, A1, A2, A3, A4, A5]. Marginal Sharpe ratios are now very different.

In this example the order of strategies by MSR is similar to the order implied by individual SR values. The only change is that A0 now becomes slightly more favorable than A1, although the individual SR of A1 is higher. The strategies A2 and A5 are negative contributors, whereas the strategies [A3, A4, A0, A1] are positive contributors to the total SR.

4 Conclusion

In order to assess new investment opportunities one needs to consider not only expected returns of new investments but also how the inclusions of new investment change the overall risk profile of the base portfolio. In this article we have shown the concept of the Marginal Sharpe Ratio (MSR) which is defined as the derivative of the portfolio Sharpe ratio over the strategy weight. MSR concept is similar to the well known concept of the marginal risk which is often used in risk management applications. We have shown the analytical derivation for MSR which leads to the very simple and intuitive formulas which can be used for quick calculations needed for appraisals of new investment opportunities. The use of the MSR formula is demonstrated with the simple numerical experiments.

5 Acknowledgment

I would like to thank Marcos Lopez de Prado, Alexandre Vladimirovitch Antonov, Matthew Rothman, Marko Sestovic and Antoine Arnoud for interesting discussions and valuable suggestions.

6 Appendix: Homogeneous functions and Euler Theorem

Multidimensional function $f : R^n \rightarrow R$ is called homogeneous of a degree k if it satisfies the equation

$$f(ax) = a^k f(x) \tag{21}$$

for all $a > 0$.

Euler theorem states that: If the function is continuously differentiable, a function is homogeneous of a degree k if and only if

$$x \nabla f(x) = k f(x) \tag{22}$$

where $\nabla f(x) = \sum_i \frac{\partial f}{\partial x_i}$ is a gradient.

This leads to a nice rule for decomposition of homogeneous functions

$$k f(x) = \sum_i x_i \frac{\partial f}{\partial x_i} \tag{23}$$

This function provide a way to express a value of a function as a sum of contributions coming from different variables x_i.

Note that risk σ_p is a homogeneous function of weights, of the order 1. If we increase leverage for a specific factor, risk will increase for the same factor. From this we conclude:

$$\sigma_p(x) = \sum_i w_i \frac{\partial \sigma_p}{\partial w_i} \tag{24}$$

Sharpe ratio is a homogeneous function of weights, of the order 0. If we increase leverage, Sharpe ratio does not change. From this we conclude:

$$0 = \sum_i w_i \frac{\partial \mathtt{SR}_p}{\partial w_i} \tag{25}$$

7 Appendix: Marginal Risk

The concept of the marginal risk (MR) is often used in Risk Management applications [10]. MR can be defined as:

$$\mathtt{MR}_i = \frac{\partial \sigma_p}{\partial w_i} \tag{26}$$

It measures how the risk of the portfolio is expected to change if we increase the weight of asset i. It takes into account diversification effects introduced by asset i.

MR can help to identify main sources of the portfolio risk. It can be used also for creating more balanced portfolios from the risk perspective [11].

Note that the risk σ_p is a homogeneous function of weights (Eq:7), of the order 1 (see Appendix 6). If we increase the leverage for a specific factor, i.e., if we multiply all weights by the same factor, the risk will increase for the same factor. As shown in the Appendix 7 we can apply the Euler theorem which leads to:

$$\sigma_p(x) = \sum_i w_i \frac{\partial \sigma_p}{\partial w_i} = \sum_i w_i \mathtt{MR}_i \tag{27}$$

MR can be easily derived analytically which yields:

$$\mathtt{MR}_i = \frac{1}{\sigma_p} \sum_j w_j \rho_{i,j} \sigma_i \sigma_j = \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p} \tag{28}$$

This can be expressed in terms of the beta of the strategy i with respect to the portfolio P

$$\beta_{i,p} = \frac{\mathtt{Cov}(r_i, r_p)}{\sigma_p^2} \tag{29}$$

which leads to

$$\mathtt{MR}_i = \beta_{i,p} \sigma_p \tag{30}$$

See the literature [10, 11] for further discussion on applications for asset allocation purposes.

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